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I. Solution by **ROBERT E. MORITZ**, B. Sc., Professor of Mathematics in Hastings College, Hastings, Nebraska; and **EDGAR KESNER**, Boulder, Colorado.

Adding and subtracting $4y^2z^2$, we have

$$(x^4+y^4+z^4-2y^2z^2-2x^2z^2+2y^2z^2)-4y^2z^2, =(x^2-y^2-z^2)^2-(2yz)^2.$$

$$\begin{aligned}(x^2-y^2-z^2)^2-(2yz)^2 &=(x^2-y^2-z^2-2yz)(x^2-y^2-z^2+2yz)\\ &=[x^2-(y+z)^2][x^2-(y-z)^2]\\ &=(x-y-z)(x+y+z)(x-y+z)(x+y-z),\\ \text{or } &-(y+z-x)(x+y+z)(x-y+z)(x+y-z).\end{aligned}$$

Similarly solved by *O. W. ANTHONY*, *J. SCHEFFER*, *C. D. SCHMITT*, *H. C. WILKES*, *B. F. YANNEY*, and *G. B. M. ZERR*.

II. Solution by **A. P. READ**, A. M., Clarence, Missouri.

By the method of the last solution, we get

$$[x^2-(y+z)^2][x^2-(y-z)^2]=(x-y-z)(x+y+z)(x-y+z)(x+y-z).$$

In a similar way by adding and subtracting first $4x^2z^2$ and then $4x^2y^2$, we obtain $(y-x-z)(y+x+z)(y-x+z)(y+x-z)$,

and $(z-x-y)(z+x+y)(z-x+y)(z+x-y)$.

Also solved in this way by *M. A. GRUBER*.

III. Solution by **ALFRED HUME**, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Lafayette County, Mississippi.

$x^4+y^4+z^4-2y^2z^2-2z^2x^2-2x^2y^2$ may be expressed as the determinant

$$\begin{vmatrix}0&1&1&1\\1&0&z^2&y^2\\1&z^2&0&x^2\\1&y^2&x^2&0\end{vmatrix}$$

which, as in Burnside and Panton's *Theory of Equations*, second edition, page 253, or, as in Weld's *Theory of Determinants*, pages 41 and 42, may be resolved into the factors

$$-(x+y+z)(y+z-x)(z+x-y)(x+y-z).$$

55. Proposed by **MARCUS BAKER**, M. A., U. S. Geological Survey, Washington, D. C.

Two right triangles ABC and ABD are so placed as to have one side $x(=AB)$ in common. From P the intersection of their hypotenuses is drawn c perpendicular to x. Knowing the hypotenuses $a=39$ feet and $b=25$ feet, and the perpendicular $c=12\frac{6}{7}$ feet, find x. Note this theorem

$$\frac{1}{m}+\frac{1}{n}+\frac{1}{c} \text{ or } \frac{1}{\sqrt{a^2+x^2}}+\frac{1}{\sqrt{b^2-x^2}}=\frac{1}{c},$$

where m and n are the altitudes of the two triangles, respectively. Also find locus of P. Discuss the case when the triangles are general (not right angled).

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Arkansas-Texas.

Let $AB=x$, $PG=c$, $AC=a$, $BD=b$, $CB=m$, $AD=n$.

From the triangles ABC and AGP, we get

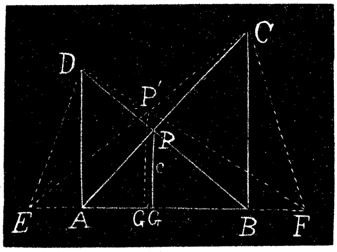
$$m:c=x:x-GB\text{...........}(1).$$

From the triangles ABD and BGP, we get

$$n:c=x:GB\text{..............}(2).$$

Eliminating GB between (1) and (2), we get

$$\frac{1}{m}+\frac{1}{n}=\frac{1}{c} \text{ or } \frac{1}{\sqrt{a^2-x^2}}+\frac{1}{\sqrt{b^2-x^2}}=\frac{1}{c}.$$



But $a=39$, $b=25$, $c=12\frac{6}{7}$.

$$\therefore \frac{1}{\sqrt{1521-x^2}}+\frac{1}{\sqrt{625-x^2}}=\frac{7}{90}. \text{ Let } 1521-x^2=y^2.$$

$$\therefore \frac{1}{y}+\frac{1}{\sqrt{y^2-896}}=\frac{7}{90}.$$

$$\therefore 7y^4-180y^3-6272y^2+161280y-1036800=0. \quad \therefore y=36,\ x=15.$$

For locus of P, let A be the origin. Using polar co-ordinates, we get

$$\tan\theta=\frac{\sqrt{a^2-x^2}}{x}, \text{ and} \frac{r\sin\theta}{x-r\cos\theta}=\frac{\sqrt{b^2-x^2}}{x}, \text{ for the equations to } AC \text{ and } BD.$$

The value of x from the first in the second gives

$$(a^2-b^2)(r\pm a)^2=(a^4\mp 2a^3r)\sin^2\theta. \quad \text{If } a=b,\ r=\pm\tfrac{1}{2}a.$$

For the general triangle, let $EF=x$, $P'G'=c$, $EC=a$, $DF=b$, $BC=m$; $AD=n$. Then from similar right triangles, we deduce the relation:

$$\frac{c}{n}(x-\sqrt{b^2-n^2})+\frac{c}{m}(x-\sqrt{a^2-m^2})=x.$$

II. Solution by Professor J. SCHEFFER, A. M., Hagerstown, Maryland, and COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Let $BC=m$, $AD=n$, $PG=c$.

Then $$m:c::AB:AG. \quad \therefore AG=\frac{c.AB}{m};$$

$$n:c::AB:BG. \quad \therefore BG=\frac{c.AB}{n}.$$

Adding, $AG+BG=AB=\dfrac{c.AB}{m}+\dfrac{c.AB}{n}$,

$$\text{or } 1=\frac{c}{m}+\frac{c}{n}, \text{ whence } \frac{1}{m}+\frac{1}{n}=\frac{1}{c}.$$

$$m=\sqrt{a^2-x^2}=\sqrt{1521-x^2}\ ;\ n=\sqrt{b^2-x^2}=\sqrt{625-x^2}.$$

Whence $\dfrac{1}{\sqrt{1521-x^2}}+\dfrac{1}{\sqrt{625-x^2}}=\dfrac{7}{90}$. Solving, $x=15$.

[SCHEFFER, SCHMITT.]

Putting $AG=x$, $PG=y$, we find from $\sqrt{b^2-\overline{AB}^2}:y=AB:x$, $\overline{AB}^2=\dfrac{b^2x^2}{x^2+y^2}$; and substituting this in $\dfrac{a^2-\overline{AB}^2}{y^2}=\dfrac{\overline{AB}^2}{(AB-x)^2}$, we get for the Cartesian equation of the locus $\dfrac{(a^2-b^2)x^2+a^2y^2}{y^2(x^2+y^2)}=\dfrac{b^2}{(b-\sqrt{x^2+y^2})^2}$.

Changing this into the polar equation by putting $x=r\cos\theta$, $y=r\sin\theta$, $x^2+y^2=r^2$, we obtain $r=\dfrac{b\sqrt{a^2-b^2\cos^2\theta}}{b\sin\theta+\sqrt{a^2-b^2\cos^2\theta}}$.

Also solved by *A. H. BELL* and *H. C. WILKES*. [See No. 24, Geometry, Vol. I, page 353, for another solution of a similar problem. Mr. Bell sends a trigonometrical solution, and says that his view of the problem in general is to have given a, b, c, and angles $ABC=BAD=\theta$, to find the base. EDITOR.]

ERRATA.—On page 359, line 16 of December issue, for "t_1+t_1" read t_1+t_2 ; page 360, line 7, in the denominator for "m_5" read m^5 ; page 360, under Case III., for "$-4m^5<n^2$" read $-4m^5>n^2$; in the last line on same page insert $\frac{1}{2}$ before the second radical; and on page 361, line 3, of problem 52, for "(2)" read (3).

PROBLEMS.

62. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

A man raises 1 chicken the first year ; 6, the second ; 35, the third ; 180, the fourth ; 921, the fifth ; 4626, the sixth ; 23215, the seventh ; 116160, the eighth ; and so on. How many does he raise the 20th year, and how many in the twenty years ?

63. Proposed by F. M. SHIELDS, Coopwood, Mississippi.

A, B, and C bought unequal shares in 200 acres of land at same price per acre, which they sold for $286.90. A gained as much per cent. on his part as he had acres, B gained $\frac{5}{8}$ as much per cent. on his part as A did, and C lost $9.10 on the cost of his part ; the total net gain was $43\frac{9}{20}$ per cent. How much land did each buy, and what did each receive per acre at the sale ?